*International Journal of Theoretical Physics, Vol. 34, No. 8, 1995*

# d-Space Problem and Solution Set: Relearning to Walk After a Stroke

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*Received October 10, 1994*

A discrete formalism, d-spaces, developed for analyzing complex movements, can be used to construct modes in which a stroke patient may relearn to walk. A specific example is sketched. The analogy between movement dissonance and quantum incompatibility is explored, along with an observationally-based distinction between dissonant and nonfunctional movement.

This paper presents a mathematical structure for biology that combines continuous and discrete aspects. Mathematical biology seems to avoid the essentially discrete or algebraic like one magnetic north pole avoids another. Mathematical biology slides off the discrete either toward small numbers, for example, by studying the trajectory taken by one arm, or toward large numbers, by taking many cells or animals to the continuum limit.

However, the interaction of several structures in the nervous system is essentially a middle-number problem. That is also true of movement problems involving the interaction of several body parts and elements of the environment.

For example, consider the movements the human organism makes at the dinner table, with knife, fork, and spoon. Each utensil is used in a different way, for a different purpose. The movements used with the three different utensils cannot just be averaged; the average would not be meaningful, because it would not accomplish any of the purposes for which the knife, fork, and spoon are used. One could meaningfully study the arm trajectories used in the movements; that is a continuous aspect of the overall dinner table movement pattern. However, the arm trajectory types do not determine the

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pattern of use of knife, fork, and spoon. There is some similarity between modules; for example, all of the spoon movements used for eating soups may be fairly similar, and for some purposes can be taken to the continuum limit by considering them to be a space of spoon movements with several continuous parameters such as speed and distance from bowl. However, spoon movements are interspersed with fork movements and knife movements in a noncontinuous way.

Another aspect of neurobiological motor control is the range of variation permitted in making each kind of movement and each part of the movement. There are measurable differences in speed, angle, and position each time the spoon is dipped into the bowl. However, there is only a certain acceptable range of speeds, the angle must allow the soup to go into the spoon (the spoon must not be upside down), and the spoon must be dipped within the bowl, not outside. So the permitted range of variation is a subset of speed $\times$ angle $\times$ position, perhaps along with other parameters.

Just as quantum mechanics can be considered to break the physical world into special subspaces by means of measurement, movement control can be considered to break the motor world into both special subspaces and their special subsets. This similarity opens the possibility of interesting comparisons in formalism. Motor control has the further task of reassembly into procedures of motor performance; if the reassembly is inappropriate, it can be called "dissonant," in analogy to incompatibility in quantum mechanics (McCollum, 1994a).

d-Spaces have been proposed for formalizing movement of this type as well as the underlying neurophysiology. A d-space (no relation to D meaning difference) is a set of regions with two relations, inclusion and contiguity. Inclusion is used to express the persistence of a physiological condition in included regions. Contiguity is used to express the ability to move between regions, and depends upon a flow, or some concept of movement in the relevant subspace. A d-space can have a repeating structure, like a crystal. An example of a crystalline d-space is a d-space showing a walk, with the repeating stepping pattern. (Crystalline d-spaces are explored in detail in McCollum 1994c.)

## PROBLEMS AND SOLUTIONS

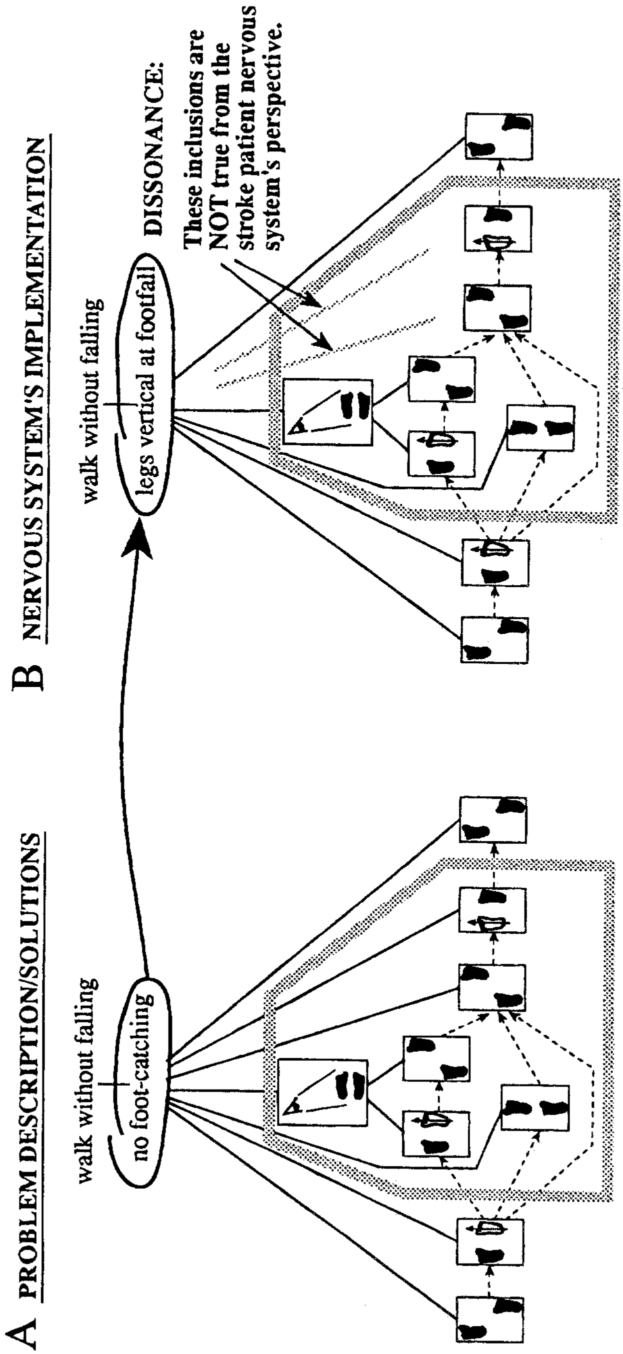
Some movement problems can be formalized by specifying starting and ending regions; solutions are movements that depart from the former and arrive in the latter. For example, a child may see a gymnast twist around a certain way. Knowing part of the movement, the child may try to perform it, guessing what was not seen. A more acute example occurs with patients recovering from neurological injuries, as in trauma or stroke. An example of a mistake made in walking during recovery from stroke was presented previously (McCollum, 1994a). The patient placed the right foot in front of the left at footfall, as if the right were the only weight-bearing foot, or as if vertical had been misconstrued. Then the left hit the right in swinging forward, so that the patient tripped.

A solution set to a problem is sort of an anticrystal, in the sense that it is nonrepeating even though each solution solves (elaborates) the same problem. Each solution specifies a sequence of regions that lead from the starting to the ending region. [For definitions and an example worked in more detail, see McCollum (1994c).] Before a solution set can be generated, (1) the including regions (invariants) must be given, along with the flows they afford, and (2) regions the patient recognizes and can use must be given. The regions used by the stroke patient can be described in terms of one foot's position with respect to the other (that is, whether the foot is staying on its own side), in terms of each leg's angle to vertical (which can include a mismatch between control assumptions versus actual performance), or in terms of the interplay between two separate movement spaces, the sagittal (forward–backward) movements of the legs and the lateral shifts of the hips over the stance foot. [This last approach is taken to the problem of putting on a jacket in McCollum (1994c).] These are probably all cogent descriptions of different aspects of function. Physiological options include teaching the patient to use another set of regions or flows, such as twisting movements. With twist, tripping might be avoided by twisting the left foot out from behind the right. Each approach requires its own notation and regions.

A sketch of a problem d-space and solution set is given by Fig. 1A. More precise notation for movement regions would be used in a comprehensive analysis. The solution set is in the grey box; the rest of the d-space is the problem. One solution—following the lowest dashed line—shows the patient learning not to cross the right foot in front of the left. The next solution shows the patient placing the right foot in front of the left, then uncrossing the feet. The feet can be uncrossed with the weight either on the left foot or on the right foot; a precise d-space would specify which. The top solution shows the imposition of an invariant: the patient watches the feet to make sure that the right foot is placed on its own side. This addition of a region above (including) all or part of a solution specifies that the nervous system will maintain that region as an invariant. Sometimes such maintenance involves a major change in the coordination of the nervous system.

## WHEN ARE SOLUTIONS THAT WORK STILL DISSONANT?

Besides the solutions given, there are many more. For example, the feet can be crossed and uncrossed an arbitrary number of times. But is that a


A PROBLEM DESCRIPTION/SOLUTIONS
walk without falling
no foot-catching
B NERVOUS SYSTEM'S IMPLEMENTATION
walk without falling
legs vertical at footfall
DISSONANCE:
These inclusions are NOT true from the stroke patient nervous system's perspective.

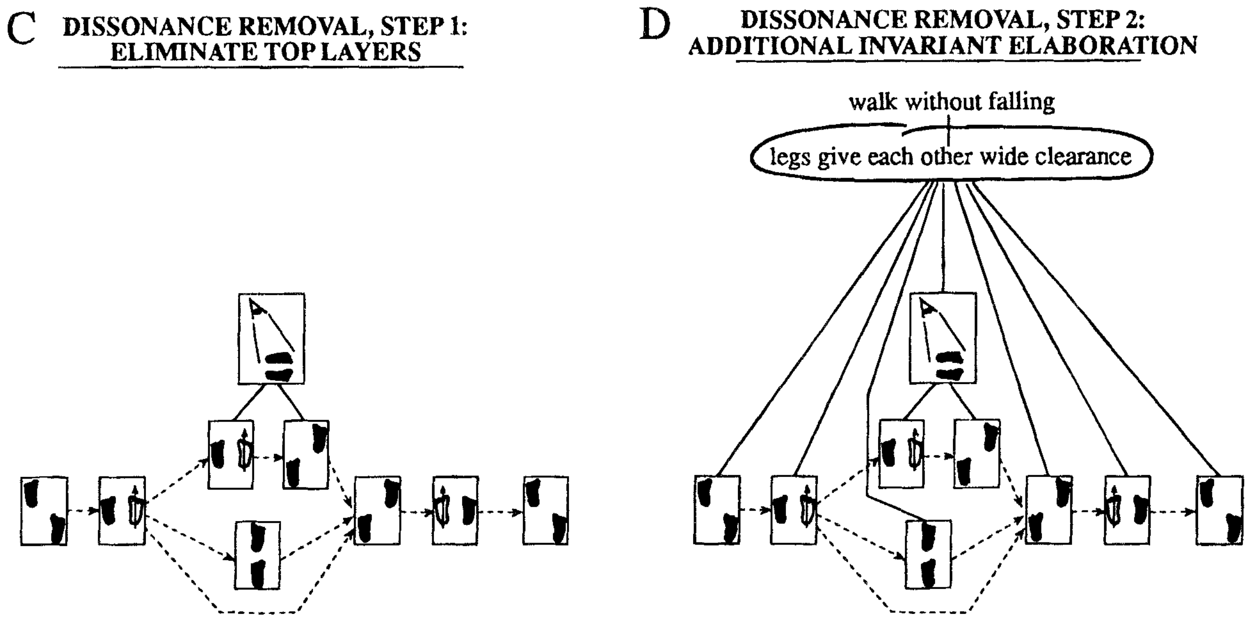



**Fig. 1.** Sketch of d-space problem and solution set. In the interest of simplicity, specific notation has not been used for the regions the particular patient recognizes and can use. Invariants are shown at the top. Included in them, arrayed across the bottom, are the regions through which the feet move. (A) The problem is stated by showing the d-space outside the grey box: the patient must start with the left foot forward (region on the bottom left), move the right foot past the left (next region, to the right), then eventually come to have the left foot forward again (rightmost region). The patient's unsuccessful behavior, placing the right foot in front of the left and tripping over it, is not shown. In the grey box are possible solutions. The lowest dashed contiguity line shows the patient simply moving the right foot forward correctly. The next contiguity up shows the patient placing the right foot in front of the left and then moving it to the right. Above that, the third solution shows the patient visually monitoring the right foot to place it correctly. (B) The patient does not implement any of these solutions as long as the invariant "no foot-catching" is construed as "legs vertical at footfall"; the solutions are wrong from the patient's point of view. (C) One way to achieve a nondissonant movement d-space is to eliminate the invariants, not easy in practice. (D) Then the invariant "legs vertical at footfall" can be replaced with another, such as "legs give each other wide clearance".

cheat? Is that like counting photons as they come out of the two-slit experiment, seeing that the diffraction pattern has been removed, and claiming that complementarity has been removed? If the patient crosses then uncrosses the feet, has the world been consistently broken into physiological parameters and reassembled into a motor procedure? What does neurophysiological consistency (nondissonance) consist of?

Mathematical aspects of d-spaces that could contribute to definition of dissonance and consistency include such requirements as that (1) flows need to alternate between components of movement such as forward–backward and side-to-side rather than moving within one component repeatedly, and (2) animals must maintain a certain proportion of height versus width in movement planning d-spaces. [For definitions of height and width, see McCollum (1994c).]

The latter formal requirement may be broken by some patients (McCollum, 1994b). Another biological clue to neurophysiological consistency/dissonance is the preference for one dominant behavioral plan. A common example is to look at your watch for one purpose—for example, to tell whether it is time to leave for a meeting at 3:45, but it turns out to be only 3:30, so the answer is 'no'—then have to look at it again just a minute later when someone asks what time it is—"tunnel memory."[2] Cats may display a similar single-mindedness when asked to walk along a horizontal ladder with a rung that sometimes descends when the cat steps on it (Andersson and Armstrong, 1987). Rather than learning to predict from forefoot experience that the rung will also be missing for the hind foot, the cat often maintains a normal walking pattern and steps on the rung, even after the experience is repeated several times.

A solution is dissonant in a down-to-earth way if it clashes with the patient's higher level perceptions. For example, the patient may perceive the right leg to be vertical at footfall only if it is crossed in front of the left. Then all of the solutions are dissonant in the sense that they no longer satisfy the patient's invariant "legs vertical at footfall" (Fig. 1B). Perhaps such a dissonance can be removed by the same route by which skills are consolidated. But first the patient would have to forget—eliminate—the original invariants and follow the region sequence of a solution by rote, the way a beginning swimmer may perform arm movements in imitation of the instructor (Fig. 1C). Only later does the combination of movements come to make sense to the learning swimmer, so that they are systematized into both hierarchy and sequence. Similarly, the patient may remove dissonance by reconstructing invariants that are consistent with the solution under them (Fig. 1D).

[2] David Finkelstein supplied the terminology.

These suggestions of the nature of dissonance are only that, suggestions. We seek to formalize an observationally based notion of biological dissonance. A good route for achieving that may be to find solution sets for problems faced by various patients, and then observe which particular solutions they actually implement, over time.

## ACKNOWLEDGMENTS

We are indebted to Patrick Roberts and David Finkelstein for discussions and suggestions.

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